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Economic Development and Gains from Trade

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1. INTRODUCTION

The interrelation of trade and long-run growth is one of the oldest issues in economic theory. Throughout the history of economic thought two opposing views on the role of trade attracted attention of academic researchers and practitioners. According to the first one, countries benefit from economic integration through efficiency-improving effects of trade on the allocation of production resources. The second view represented, in particular, by the doctrine of uneven development emphasizes the increasing divergence of per capita incomes as a result of international division of labor and long-term specialization of countries in trade. Some countries become losers because of unfavorable specialization impeding industrialization and hindering utilization of positive external effects that development might bring to less developed nations in autarky.

Generally, the problem of openness and global development includes two different issues. The first issue is about differences in economic performance. Does trade in goods, services and factors lead to a convergence in per capita incomes or it increases a tendency for uneven development? The second issue is about long-term welfare gains and losses from trade. Do all countries benefit in the long run from trade in terms of social welfare if international barriers to trade are removed?

A narrower issue addressed in this paper is about the effects of trade on the long-run performance of a country with absolute advantages in the extracting sectors. Will the resource-abundant country inevitably remain a supplier of raw materials, and to what extent can it benefit from trade? In particular, this is an important issue from the perspective of Russia's economic integration with other economies located geographically close.

Economic literature does not provide an unambiguous answer to these questions, while suggesting a diversity of "pessimistic" results. For instance, the disparity of growth has been demonstrated in the "learning-by-doing" and human capital accumulation models of economic integration (e.g. Boldrin and Sheinkman, 1988; Lucas, 1988; Young, 1991; Stokey, 1991). Endogenous growth models with R&D competition and international trade in goods may display a similar property. Feenstra (1996) utilizes the framework of Helpman and Grossman (1991) and shows that in the absence of technological diffusion trade between countries of different size may cause the divergence of incomes.

The ambiguity of theoretical results about interrelation of trade and growth to some extent reflects a variety of views on the essentials of

economic development. This process can be viewed as an accumulation of country-specific factors of production, which make national industries competitive in the global division of labor. The engine of growth in the models of endogenous growth is investment in physical or human capital, and accumulation of fundamental or technological knowledge. Importantly, factors like knowledge, blue-prints, and production equipment are tradable or transferable across countries. By this reason a broader view on openness comprising not only exchange in goods, but also in capital, ideas, and various spillover effects of R&D may not receive a convincing answer from many of these theoretical models. Broadly defined, openness would lead to an intensive international factor migration precluding accumulation of country-specific advantages and making divergence of growth hardly possible. As was shown by Feenstra (1996), the assumption about international diffusion of knowledge may play a decisive role in the demonstration of convergence effects of international trade.

In our view, accumulation of country-specific advantages is impossible without acquisition of new skills by qualified labor. This is, for instance, an essential feature of development in the transition economies where business-practices like corporate governing, marketing, book-keeping, financial and risk management, etc had been unknown or completely forgotten before the market reforms started.

Accumulation of skills has been treated in the "learning-by doing" models of trade and growth by Stokey (1991) and Young (1991), as an improvement of the average level of skills. The diversity of skills that a nation possesses also plays an important role in development and constitutes a specific factor of production, which is truly non-tradable. Unlike ideas and knowledge, skills cannot easily spill over national borders since labor is a relatively immobile factor. By different reasons barriers to international labor migration are especially pronounced for transition or developing economies. Therefore, creation of new skills in these countries requires domestic investment or individual efforts in the form of schooling, learning, training, reeducation, *etc.*, which induce a raise in the national level of economic development.

Accumulation of skills may exert a significant external effect on production. Acquisition of skills is normally a household decision that does not internalize completely marginal effects of new skill creation on production and trade. Therefore, the number and diversity of skills constitute a factor of production inadequately priced by the market. The external effects of skill creation may, to a large extent, shape the process of economic development.

This view on industrial development is reflected by the model presented here, which focuses on the long-term effects of international trade on the resource-rich country. The model describes trade between two asymmetric countries endowed with three primary production factors: skilled labor, unskilled labor, and a stock of a natural resource. For simplicity, all these factor endowments are assumed constant in time. The natural resource is thus inexhaustible and the word "stock" in the case of our model refers to the impact of "nature" on national output. The number of skills is a time-varying endogenous factor that determines dynamic comparative advantages of nations in the bilateral trade. Accumulation of skills in the model is a result of private investment decisions made by households. To make the model analytically tractable we abstract from physical capital accumulation.

Each economy consists of two competitive production sectors: a downstream manufacturing sector, and an upstream traditional sector. Skilled labor serves in the production of the final manufacturing good. Unskilled labor and the natural resource are used for the production of the raw material by the upstream sector. The raw material is an intermediate input for the manufacturing sector. Final manufactured goods produced in both countries are perfect substitutes in consumption and can be exchanged for the raw material in bilateral trade.

The diversity of skills has a positive impact on manufacturing because skills are imperfect substitutes in production. Horizontal differentiation of skills plays the same role in our model, as horizontal differentiation of intermediate inputs in the models of endogenous growth (Grossman & Helpman, 1991). To make clear the economic intuition, we can suggest quite a broad interpretation of skills as a fringe of differentiated intermediate producer services. These, in line with Ciccone and Matsuyama, include "equipment repair and maintenance, transportation and communication services, engineering and legal supports, accounting, advertising, and financial services, and so on" (Ciccone & Matsuyama, 1996, p. 34). Understood like this, the number-of-skill model is not very different from the variety-expanding endogenous growth models.

Household preferences and production technologies in the two countries are symmetric. The asymmetry appears only in households' endowments. One of the countries is supposed resource-abundant: it has a larger stock of the natural resource and, hence, possesses an absolute advantage in the production of the raw material. Furthermore, the countries are endowed with different numbers of skills. Comparative advantages in trade are determined at each instant by the relationship between cross-countries ratios of the numbers of skills and the volumes of the natural resource.

The country with comparative advantage in manufacturing imports the raw material and exports the manufacturing good. The model demonstrates that if returns on skills are decreasing, development paths of the countries converge. Under decreasing returns the resource-abundant country will eventually specialize in exporting the raw material, no matter how it is developed initially. Even if it has initially a comparative advantage in manufacturing and imports the raw material from the resource-scarce country, it loses this advantage in the course of transition to the steady state. This is a result of convergence of returns on skills to the discount rates identical across countries.

Both countries gain from trade in the short run but the resource-abundant country may lose from an unfavorable development pattern. It tends to under-accumulate skills as compared to the case of autarky. The resulting long-term effect of trade on social welfare is positive for the resource-scarce country and may be negative for the resource-abundant country. The latter is a loser if the external effect of skills on production is pronounced or, equivalently, various skills are poor substitutes. The resource-abundant country gains from trade if this external effect is insignificant.

The situation is different in the case of increasing returns on skills. In this case economic dynamics are indeterminate: the initial endowments of skills are not sufficient to define equilibrium paths. The long-run evolution of the economy depends among other factors on self-fulfilling initial beliefs. Therefore comparison of openness regimes in our model faces some methodological difficulties. For instance, it is not completely clear which trading equilibrium paths are to be compared with autarky equilibrium paths. Nevertheless, the model provides some arguments that trade under increasing returns bolsters the divergence of growth. The reason is that returns on skills do not converge to the discount rate and may remain in the long run different between countries. Welfare comparison shows that households in the resource-abundant country gain from trade under increasing returns. They reduce consumption at the early stage of development and increase investment in skills. The trading equilibrium path provides therefore a higher long-run number of skills and a higher long-run welfare in the resource-abundant country than the autarky equilibrium path.

The paper is organized as follows. The next section introduces the model; Section 3 analyzes the autarky equilibrium, and Section 4 examines the trading equilibrium. Section 5 provides an analysis of convergence and Section 6 addresses the welfare issue. Finally, conclusions are drawn in Section 7.

2. THE MODEL

2.1. Households

The economy is populated by infinitely living homogenous households. The size of the population is constant over time and normalized to unity. Households are endowed with three primary factors: skilled labor, unskilled labor, and a stock of a natural resource. These are exogenous production factors that are assumed constant in time. The total number of skills supplied by households is an endogenous production factor. The representative household chooses a consumption path and invests in the accumulation of skills. It does not influence production decisions by firms and expected returns on skills. At each instant the household inelastically supplies a unity of skilled labor and a unity of unskilled labor to the firms.

The household's problem is maximizing its discounted utility function

$$\int_0^{\infty} e^{-\delta t} u(c) dt, \quad (1)$$

subject to

$$\dot{n} = y - c, \quad (2)$$

$$\dot{n} \geq 0. \quad (3)$$

Here y is per capita income, c is consumption, δ is the discount rate, n is the current number of skills. Equation (2) is a dynamic budget constraint. Creation of a new skill requires a unity of final output. According to (3) individuals cannot sell skills that they have acquired. The instantaneous consumption utility $u(c)$ is strictly concave and continuously differentiable.

2.2. Production

There are two production sectors in the economy – manufacturing and extracting. The final manufacturing sector is populated with homogenous and competitive firms operating under constant returns to scale. The number of firms is constant and normalized to unity. Firms select inputs of skilled labor and the raw material supplied by the extracting

sector. The manufacturing firm's production function is

$$y = L^\alpha R^{1-\alpha}, \quad (4)$$

where

$$L = \left\{ \int_0^n l(i)^{1/\theta} di \right\}^\theta$$

is a composite skilled labor input, and R is the raw material input. Here $l(i)$ denotes the labor input of Skill i , $\theta = \sigma/(\sigma - 1)$, and σ is the partial elasticity of substitution. Skills are imperfect substitutes, $\sigma > 1$, and parameter $\theta \geq 1$ measures the differentiation of skills.

The manufacturing firm chooses inputs to maximize instantaneous profit:

$$\pi = y - \int_0^n w(i)l(i)di - pR,$$

where $w(i)$ is the price of skill i , p is the raw material price.

The extracting sector consists of competitive and identical firms. The number of firms in the extracting sector is constant and normalized to unity. The extracting firm exploits the stock of the natural resource, N , and the unskilled labor input, l_u . The production function of the extracting firm, $R = f(N, l_u)$ is homogenous.

3. THE AUTARKY EQUILIBRIUM

Suppose that the economy is closed. The labor markets clear if $l = 1/n$, and $l_u = 1$. Skills enter production function (4) symmetrically, and $l(i) = l$ for all i . Therefore we omit here and henceforth the index of skills, when possible. The composite labor input is $L = l \cdot n^\theta = n^{\theta-1}$. Aggregate supply is then determined by the number of skills and the raw material supply:

$$y(n, R) = n^{\alpha(\theta-1)} R^{1-\alpha}. \quad (5)$$

Production by the extracting firm is constant in time: $R = f(N, 1)$.

The first-order conditions for the final good production are

$$\alpha(L/R)^{\alpha-1} \partial L / \partial l = w$$

and

$$(1 - \alpha)(R/L)^{-\alpha} = p.$$

One can easily show that¹

$$\frac{\partial L}{\partial l} = n^{\theta-1}.$$

Factor prices are determined as:

$$w = \alpha n^{\alpha(\theta-1)} R^{1-\alpha} = \alpha y(n, R), \quad (6)$$

$$p = (1 - \alpha) n^{\alpha(\theta-1)} R^{-\alpha} = (1 - \alpha) y(n, R) / R. \quad (7)$$

Unskilled labor is rewarded according to its share in the raw material production. In fact, we do not need to specify this share, or the share of the natural resource in household income.

The individual makes current consumption-investment decision by comparing instantaneous return on the new skill with the discount rate. By assumption, the individual does not influence the return on skills. To define this return properly, we have to use a notational difference for individual and economy-wide number of skills. Let n denote the former and n' the latter. In fact the number of skills that appears in the firm production function (4) relates to the economy-wide number of skills. The single skill input l and equilibrium output $y(n', R)$ are also determined by n' . Hence, the return on the skill can be represented as a function $r(n') = w(n')\iota$, where ι is the individual single skill input. By symmetry of skills and homogeneity of individuals, $\iota = l = 1/n'$.² The total skill premium is $w = r(n')n$, and the household income is linear in n : $y = r(n')n + pR$. The marginal return on skills is thus $r(n') = \alpha y(n', R)/n'$. In what follows we do not use the symbol of prime in notation of this variable.

We thus ensure concavity of the household problem, irrespective of the kind of dependence between the marginal return and the number of

¹ Indeed,

$$\partial L / \partial l(i) = \theta \left(\int_0^n l(i)^{1/\theta} di \right)^{\theta-1} \theta^{-1} l(i)^{1/\theta-1}.$$

Inserting $l(i) = 1/n$ yields $\partial L / \partial l = n^{\theta-1}$.

² One can suggest a probabilistic interpretation for the single skill intensity ι . It may be regarded from the individual's view as an instantaneous probability of being employed by the firm at some position corresponding to the specific skill.

skills. The returns on skills are decreasing if

$$\frac{dr}{dn} = \alpha \frac{\partial(y(n, R) / n)}{\partial n} < 0,$$

or $\alpha(\theta - 1) < 1$, that is the elasticity of substitution between different skills is quite high, $\sigma > 1 + \alpha$. The returns are increasing if the elasticity of substitution is low, $\sigma < 1 + \alpha$.

Proposition 1. Equilibrium dynamics under autarky satisfies the system

$$\dot{n} = y(n, R) - c, \quad (8)$$

$$\dot{c} = h(c)(\alpha y(n, R) / n - \delta), \quad (9)$$

where $h(c) = -u'(c)/u''(c)$ is reciprocal to the intertemporal substitution rate. The steady state of this system (n^a, c^a) is unique. It is a saddle point if returns on skills are decreasing, and an unstable spiral or node if returns are increasing.

According to Proposition 1, the model exhibits two cases of equilibrium dynamics. The first one corresponds to decreasing returns on skills and reproduces the pattern of neoclassical growth. Given any initial range of skills n_0 such that $n_0 < n^a$, the equilibrium path is a saddle trajectory converging to the stationary point $S = (n^a, c^a)$. The equilibrium path is drawn with bold line in Fig. 1, which illustrates the phase space of the system (8), (9). The stationary point S is a unique autarky steady state of the model. The saddle equilibrium trajectory satisfies any conventional transversality condition. The number of skills n is monotonously

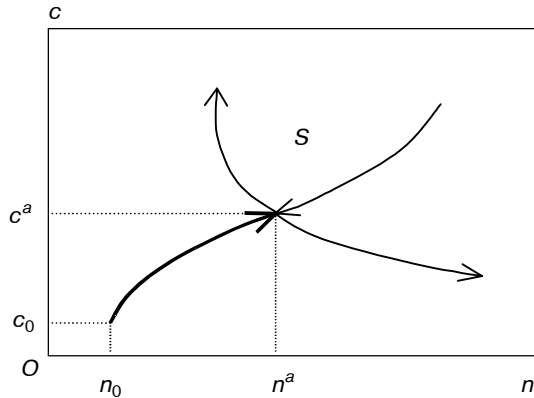


Figure 1. The autarky equilibrium path: decreasing returns.

increasing along the equilibrium path, and the constraint (3) of the household problem is non-binding at any instant.

The second case of equilibrium dynamics emerges when returns on skills are increasing. The phase space is depicted in Fig. 2. The stationary point S of the dynamic system is unstable and neither trajectory in the phase space achieves this point.

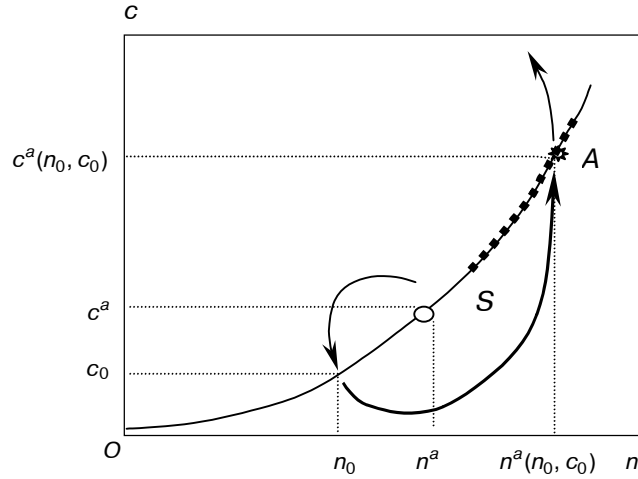


Figure 2. The autarky equilibrium path: increasing returns.

The curve OA is a locus of zero investment such that $c = y(n, R)$. Any equilibrium trajectory intersects this curve at some finite point in time when constraint (3) of the household problem becomes binding. At this and subsequent moments consumption equals current income because households cannot sell skills to increase consumption. The intersection point is thus a steady state of the model under the binding investment constraint. Households do not invest in skills in this state and consumption remains constant in time.

The long-run development under increasing returns on skills is indeterminate. Given any initial number of skills, n_0 , the equilibrium path is defined by the choice of initial consumption c_0 among a continuum of feasible values of initial consumption located below locus OA in Fig. 2. Each equilibrium path uniquely determines a zero-investment steady state, which is located in a domain belonging to locus OA and marked with bold dots. Individual initial consumption is determined by the first order condition to the household problem (1) – (3) equating marginal

utility of consumption and the value of new skill (see proof of Proposition 1 in Appendix). The initial value of skills is, hence, determined on social basis, and the initial individual choice of consumption depends on individual choices by all households. This per se leaves room for multiplicity of equilibrium paths and self-fulfilling beliefs about long-run development and steady-state consumption.

Point $A = (n^a(n_0, c_0), c^a(n_0, c_0))$ in Fig. 2 is an example of steady state under increasing returns determined by an initial number of skills equal to n_0 and initial consumption c_0 . An equilibrium trajectory starting from this initial point is drawn with bold line. For any initial number of skills satisfying $n_0 < n^a$, the equilibrium consumption path is decreasing at the beginning of transition. Once n exceeds n^a , consumption starts to increase until the steady state number of skills $n^a(n_0, c_0) > n^a$ is reached. The steady-state number of skills and consumption level under increasing returns is higher than under decreasing returns. Dynamics of consumption is thus *J*-shaped: the return on skills is below the discount rate at the first stage of development and above this rate at the second stage. The return on skills is increasing in time and exceeds the discount rate at any stationary state $(n^a(n_0, c_0), c^a(n_0, c_0))$.³

4. THE TRADING EQUILIBRIUM

Consider a two-country world with bilateral trade in goods. All production factors including skills are immobile. The countries are identical in terms of population size, labor endowments, preferences and technologies, but differ in the size of the natural resource and the number of skills.

Trade is free, and prices of traded goods are equalized at each instant. Due to linear homogeneity of production, unit costs in both sectors are also equalized, and complete specialization in production is ruled out. The country with comparative advantage in manufacturing imports the raw material and exports the final good. Trade is balanced at each instant.

Let X_j denote the raw material input in manufacturing in country j . Here and henceforth subscript $j = 1, 2$ labels country. Let ΔR be the volume of the raw material exported (imported if $\Delta R < 0$) by country 1. Then

³ We did not analyze the case of constant returns on skills, when $\sigma = 1 + \alpha$. In fact, the behavior of equilibrium paths in this case is qualitatively close to the one determined under increasing returns.

$X_1 = R_1 - \Delta R$, and $X_2 = R_2 + \Delta R$. Since trade is balanced, country 1 imports (exports if $\Delta R < 0$) $p\Delta R$ units of the final good where p is the raw material price. Household disposable income is $q_1 = y_1 + p\Delta R$ in country 1, and $q_2 = y_2 - p\Delta R$ in country 2. We assume that country 2 is resource-abundant and country 1 is resource-scarce: $N_2 > N_1$ and, hence, $R_2 > R_1$, and denote the ratio R_2/R_1 as ρ . This ratio determines an absolute advantage of country 2 in the extracting sector.

The household problem under free trade is:

$$\int_0^{\infty} e^{-\delta t} u(c_j) dt, \quad (10)$$

subject to

$$\dot{n}_j = q_j - c_j, \quad (11)$$

$$\dot{n}_j \geq 0, \quad (12)$$

$j = 1, 2$. The labor market clears in each country if $l_j = 1/n_j$, $l_{uj} = 1$. Like in the autarky case, equilibrium final output in country j is

$$y(n_j, X_j) = n_j^{\alpha(\theta-1)} X_j^{1-\alpha}, \quad (13)$$

and the raw material production is $R_j = f(N_j, 1)$.

The first-order conditions for profit maximization by the manufacturing firm are similar to (6), (7) and imply that

$$p_j = \frac{(1-\alpha)w_j}{\alpha X_j}.$$

The equalization of prices is reached at each instant through an adjustment of relative wage w_2/w_1 and trade volume ΔR . The raw material prices are equalized, $p_1 = p_2 = p$, if the following condition holds

$$\frac{w_2}{w_1} = \frac{X_2}{X_1}.$$

Similarly, the final good prices are equalized if unit costs in manufacturing:

$$z_j = (w_j + p_j X_j) / y(n_j X_j) = n_j^{-\alpha(\theta-1)} X_j^{\alpha-1} w_j / \alpha$$

are identical across the countries. Note that this cost is normalized to unity in one of the countries and equals to unity in the other. Equaliza-

tion of unit costs implies that

$$\frac{w_2}{w_1} = \left(\frac{n_2}{n_1} \right)^{\alpha(\theta-1)} \left(\frac{X_2}{X_1} \right)^{1-\alpha}.$$

Consequently, the following conditions must hold under free trade:

$$\frac{w_2}{w_1} = \frac{X_2}{X_1} = \left(\frac{n_2}{n_1} \right)^{\theta-1}. \quad (14)$$

These equations determine the terms of trade and the trade volume.

Proposition 2. The trade volume is

$$\Delta R = R_1 \frac{(n_2/n_1)^{\theta-1} - \rho}{1 + (n_2/n_1)^{\theta-1}}.$$

According to Proposition 2, if country 1 has a comparative advantage in extraction, that is $(n_2/n_1)^{\theta-1} > \rho$, it exports the amount ΔR of the basic good to country 2, and vice-versa. The country with a higher number of skills has an absolute advantage in manufacturing but does not necessarily possess a comparative advantage in manufacturing. The higher the external effect of skill creation on manufacturing indicated by parameter θ , the lower the level of absolute advantage in skills required for comparative advantage in manufacturing.

Corollary. The inputs of raw material are

$$X_j = \frac{R_1 + R_2}{1 + (n_i/n_j)^{\theta-1}}.$$

The household incomes are

$$q_j = y(n_j, X_j)(\alpha + (1-\alpha)R_j/X_j) \equiv q_j(n_j, X_j),$$

$j = 1, 2$. As in autarky, individuals make investment decisions comparing the return on skills with the discount rate. They ignore the effects of new skill creation on the manufacturing firms' production technology. They also ignore the effects of investment on the terms of trade and the volume of trade. Therefore, as above, marginal return on the new skill equals skill premium $r_j = \alpha y(n_j, X_j)/n_j$, and it does not include the return on international exchange.

The raw material input X_j depends on the numbers of skills in both countries. An increase of the number of skills in a country raises output

through two effects: directly, through labor productivity growth, and indirectly, through an increase in the country's raw material input. Therefore, unlike the case of autarky, the return on skills is by definition decreasing (increasing) in the number of skills if:

$$\frac{\partial r_j}{\partial n_j} = \alpha \partial [y(n_j, X_j(n_j, n_i)) / n_j] / \partial n_j < (>) 0,$$

where $X_j(n_j, n_i)$ is the raw material input as a function of the numbers of skills.

Proposition 3. Equilibrium dynamics under free trade satisfies the system

$$\dot{n}_j = q_j(n_j, X_j) - c_j, \quad (15)$$

$$\dot{c}_j = h(c_j)(\alpha y(n_j, X_j) / n_j - \delta), \quad (16)$$

$j = 1, 2$. This system has a unique stationary state $(n_j^*, c_j^*)_{j=1,2}$. It is a saddle point with two negative characteristic roots if the return on skills is decreasing at the steady state, or $\sigma > (3 + \alpha)/2$.

Stable saddle trajectories in the case of decreasing returns belong to a manifold of dimension two and converge to the stationary saddle point (n_j^*, c_j^*) . Given any combination of initial conditions (n_{01}, n_{02}) , an equilibrium path under decreasing returns is determined by the choice of initial consumption for both countries (c_{01}, c_{02}) . If initial conditions are such that $n_{0j} < n_j^*, j = 1, 2$, the equilibrium path is a saddle trajectory of the system (15), (16) converging to the stationary state $(n_j^*, c_j^*)_{j=1,2}$. We will characterize this state in the next section.

The steady state (n_j^*, c_j^*) has one or none of negative real-value roots in the case of increasing returns.⁴ Therefore this steady state is unattainable for an arbitrary pair of initial conditions. Given such a pair (n_{01}, n_{02}) , one cannot select a set of initial consumption values, (c_{01}, c_{02}) , that ensures convergence of the equilibrium trajectory to the saddle steady state (n_j^*, c_j^*) . As in autarky, equilibrium dynamics under

⁴ In fact, the necessary condition that the steady state (n_j^*, c_j^*) have two negative roots is that the intercept of characteristic polynomial is positive, that is $\sigma > 1 + \alpha$ (see proof to Proposition 3).

increasing returns are qualitatively different due to the indeterminacy of equilibrium paths. Each of these paths attains a stationary point satisfying zero-investment conditions $q_j(n_j, X_j) = c_j$, $j = 1, 2$. These points constitute multiple steady states of the model determined by the initial choices of consumption.

5. THE PATTERN OF TRADE AND DEVELOPMENT

Incomes in the two countries diverge under autarky because the economies differ in endowments of the natural resource. In fact, the long-run number of skills in autarky and under decreasing returns is a power function of the extracting capacity R , as seen from the steady-state condition equating return on skills to the discount rate. Therefore the divergence of growth is predetermined by the difference between countries in the natural resource endowments. The question addressed in this section is about the impact of trade on the divergence of development. Does trade in goods enhance divergence, or the countries tend to converge due to some features of economic exchange? To answer this question we have to treat the cases of decreasing and increasing returns separately because of the essential differences in equilibrium dynamics. The long run trading equilibrium under decreasing returns on skills is characterized in the next proposition.

Proposition 4. Under decreasing returns the numbers of skills and the final outputs are equalized in the long run across countries,

$$n_1^\tau = n_2^\tau, \\ y(n_1^\tau, X_1^\tau) = y(n_2^\tau, X_2^\tau).$$

The long-run consumption is higher in the resource-abundant country:

$$c_2^\tau = c_1^\tau \frac{\alpha(1+\rho) + (1-\alpha)2\rho}{\alpha(1+\rho) + (1-\alpha)2}.$$

Proposition 4 implies convergence of the countries in terms of the number of skills and manufacturing output. Whatever is the initial difference in skills between the countries, it vanishes in the long run. Nevertheless, trade is sustained because the countries differ in the natural resource endowment. The long-term trade volume is $\Delta R = (R_1 - R_2)/2 < 0$, hence the raw material input is evenly allocated between the countries:

$$X_1^\tau = X_2^\tau = \frac{R_1 + R_2}{2}.$$

The skilled labor compensations are also equalized, due to the terms of trade condition (14). Final outputs are equalized because factor inputs in the long run are identical across the countries. The resource-abundant country will ultimately specialize in exporting the raw material, no matter how it is developed initially. Even if it had initially a comparative advantage in manufacturing, $(n_{02}/n_{01})^{\theta-1} > \rho$, and imported the raw material from country 1, it loses this advantage in transition to the steady state. Country 1 reaches sooner or later a comparative advantage in manufacturing and retains this position in trade forever.

Convergence of development results from the symmetry of manufacturing technology and the tendency of returns on skills to equalize. These returns tend to the discount rates, which are identical in the two countries and decrease in time in both countries. Convergence of returns is reached through the convergence of raw material inputs X_j that change in time as

$$\dot{X}_j = (\theta - 1) \frac{X_i X_j}{R_1 + R_2} \left(\frac{\dot{n}_j}{n_j} - \frac{\dot{n}_i}{n_i} \right), \quad (17)$$

and $j, i = 1, 2$.⁵ Equation (17) indicates that the raw material input is increasing in the country with a higher current growth rate and decreasing in the other country. Suppose, for instance, that country 1 has an absolute advantage in manufacturing, that is $n_1 > n_2$, near the steady state (n_j^*, c_j^*) . It follows that the numbers of skills across countries are converging if n_1 is growing slower than n_2 . The returns on skills are converging if the raw material input X_1 is decreasing (X_2 is increasing). Thus, the country with an absolute disadvantage in skills eventually increases the raw material input and catches up the advanced country.

Convergence of development under decreasing returns on skills does not imply equalization of consumption. According to Proposition 4 household consumption in the resource-abundant country exceeds consumption in the resource-scarce country. The ratio of long-run consumption streams c_2^*/c_1^* is positively linked to the absolute advantage in extracting, ρ , and to the share of intermediate input in manufacturing output, $1 - \alpha$.

⁵ One can easily show that

$$\frac{\partial X_j}{\partial n_j} = (\theta - 1) \frac{X_i X_j}{(R_1 + R_2) n_j}, \text{ and } \frac{\partial X_j}{\partial n_i} = -(\theta - 1) \frac{X_i X_j}{(R_1 + R_2) n_i}.$$

In the case of increasing returns the stationary state $(n_j^\tau, c_j^\tau)_{j=1,2}$ is unattainable by equilibrium paths. Besides, we cannot calculate the steady-state numbers of skills achieved by these paths within finite time. To examine impact of trade on growth we consider instead transition dynamics and compare differences in growth rates between the countries under trade and autarky. We use an indicator of divergence based on marginal growth rates:

$$\Delta^s = \left(\frac{\partial \dot{n}_1}{\partial n_1} - \frac{\partial \dot{n}_2}{\partial n_2} \right)^s.$$

Here superscript s relates to autarky or trade, $s = a$ or τ . The marginal growth rate in country j represents the change in investment in skills caused by a change in the number of skills in this country. Both in autarky and trade, marginal growth rate is increasing in the number of skills due to increasing returns on skills (marginal growth rates are equal to social returns on skills, $\partial y_j / \partial n_j$ and $\partial q_j / \partial n_j$, respectively, that are increasing in n_j).

The indicator of divergence Δ^s characterizes a difference in marginal growth rates between countries for each regime. We define that trade favors growth in country 1 if $\Delta^\tau > \Delta^a$, and in country 2 if $\Delta^\tau < \Delta^a$, given that the number of skills in each country is the same under both regimes.

Proposition 5. Trade favors growth in the country with comparative advantage in manufacturing.

Proposition 5 implies that trade under increasing returns favors the country having comparative advantage in manufacturing, thus aggravating the tendency to diverge that prevails under autarky. This is so because the returns on skills in trading equilibrium do not tend to equalize across countries. The advanced country may accelerate accumulation of skills and increase the raw material input thus enhancing the disparity of growth. Such a positive feedback can work because the returns on skills do not decrease as their number grows. The economies do not converge in terms of intermediate inputs, and the numbers of skills do not equalize as the economies tend to long-run steady states.

The indicator of divergence based on marginal growth rates Δ^s characterizes an extra growth capacity that the advanced country enjoys under increasing returns on skills. The resulting long-term effect of trade on growth depends on consumption paths selected by households in each country. By this reason, the resource-abundant country may ultimately

benefit from trade even remaining a supplier of raw material in the long run. The next section addresses this issue.

6. GAINS AND LOSSES FROM TRADE

In this section we examine effects of trade on the long-run social welfare. Opening up trade between the countries influences welfare through two effects: the short-term effect on production and the long-term effect on accumulation of skills. Production effect of trade results from the reallocation of raw material from a less productive user to a more productive one. By this reason trade affects positively current incomes in both countries, as the following proposition states.

Proposition 6. Given a number of skills, instantaneous incomes in both countries are higher under free trade than under autarky:

$$q_j(n_j, X_j) > y(n_j, R_j)$$

for $j = 1, 2$. The short-term production effect of trade is positive for both countries under decreasing and increasing returns. As it can be seen from Corollary to Proposition 2, income can be represented as a product of output under autarky $y(n_j, R_j)$ and a static gain from trade $\phi(X_j/R_j)$:

$$q_j(n_j, X_j) = y(n_j, R_j) \phi(X_j/R_j)$$

where $\phi(X_j/R_j) = (X_j/R_j)^{1-\alpha}[\alpha + (1-\alpha)(R_j/X_j)] > 1$, $j = 1, 2$. Static gain from trade is a function of the ratio X_j/R_j with global minimum at the no-trade point, $X_j = R_j$. Static gain from trade is increasing, $\phi'(X_j/R_j) > 0$, for the country with comparative advantage in manufacturing, that is, for the country with $X_j > R_j$. Likewise, it is decreasing for the country with comparative advantage in extracting, when $X_j < R_j$. Any increase of trade volume causes an increase of the ratio X_j/R_j for the former country, and a decrease for the latter. Hence, incomes in both countries are rising with the volume of trade.

The long-term effect of trade on development may be opposite to the short-term effect. Convergence of development under decreasing returns and production gains from trade do not guarantee that both countries benefit in terms of accumulation of skills. The long-term gains and losses from trade are revealed by comparison of the long run numbers of skills under autarky and trade.

Proposition 7. Under decreasing returns on skills, the long-run numbers of skills under trade and autarky are related as follows:

$$n_1^T = \left(\frac{1+\rho}{2} \right)^\beta n_1^a; \quad n_2^T = \left(\frac{1+\rho}{2\rho} \right)^\beta n_2^a,$$

where $\beta = (1 - \alpha)/(1 - \alpha(\theta - 1)) > 0$. The resource-scarce country over-accumulates skills as compared to the autarky, because $n_1^T > n_1^a$ and the resource-abundant country under-accumulates skills, $n_2^T < n_2^a$. The resource-abundant country, hence, loses from trading with the resource-scarce country in terms of development. Proposition 7 also states that the greater the absolute advantage of country 2 in extracting, ρ , the higher the loss of this country and, correspondingly, the gain of country 1.

Fig. 3 illustrates Propositions 4 and 7. It depicts the equilibrium trajectory in plane $(0, n_1, n_2)$. Point *A* indicates the long-run numbers of skills under autarky, and ray *OA* indicates the long-run proportion of skills such that $n_2^a / n_1^a = \rho^\beta$. Point *T* denotes the long-run trading equilibrium. Arrow-marked curves represent equilibrium paths under autarky and trade.

Proposition 6 implies that the short-term effect of trade on production is positive for both countries. According to Proposition 7, the long-term

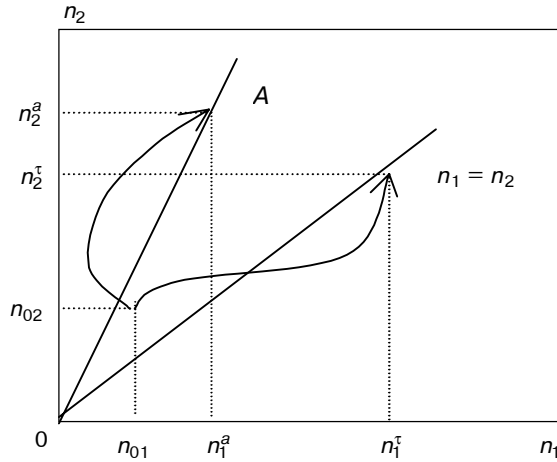


Figure 3. The effect of trade on development in the case of decreasing returns.

effect of trade on development is positive for the resource-scarce country and negative for the resource-abundant country. Intuitively, the resulting effect of opening up trade on social welfare is positive for the former and ambiguous for the latter. We compare the steady-state consumption for both countries in the next proposition.

Proposition 8. Trade under decreasing returns improves long-run welfare in the resource-scarce country. The resource-abundant country loses in terms of long-run welfare if $\sigma < 2$. This country gains from trade if $\sigma > 2$, and the following condition holds:

$$\alpha + (1 - \alpha) \frac{2\rho}{1 + \rho} > \left(\frac{2\rho}{1 + \rho} \right)^\beta. \quad (18)$$

According to this proposition, if the elasticity of substitution σ is below the critical value 2, the resource-abundant country incurs long-term welfare losses from trade. A positive static effect of trade on production is eliminated for this country by a negative dynamic effect of under-accumulation of skills. This dynamic effect is more pronounced when the elasticity of substitution is sufficiently small or, equivalently, the external effect of skill creation on production is significant. The resource-abundant country gains from trade if the external effect of development is insignificant, and this country has a high absolute advantage in extracting. This is seen from (18): if $\sigma > 2$, then $\beta < 1$, and (18) holds if ρ is high.

Welfare comparison in the case of increasing returns requires criteria for selection among multiple equilibrium paths. The question is which trading equilibrium path is to be selected for comparison with the autarky equilibrium path? Since our goal is to compare long-run consumption under autarky and trade, it makes sense to select equilibrium paths under different regimes with either the same terminal number of skills or the same initial consumption.

Suppose we can choose a trading equilibrium path that ensures the same steady-state numbers of skills in each country as in autarky. Then the steady-state consumption will be higher for both countries under trade than under autarky, as follows from Proposition 6. In other words, if each country has the same long-run number of skills in trade and in autarky, both countries gain from trade in terms of long-run welfare.

Suppose now that the trading equilibrium trajectory starts from the same initial consumption as the autarky equilibrium path. To simplify welfare comparison in this situation, suppose that the resource-abundant country has a comparative advantage in extracting along the

whole equilibrium path. This path is compared with the autarky path governed by system (8), (9).

Proposition 9. Suppose the utility function, $u(c)$, has a constant rate of intertemporal substitution, $h(c) = \text{const}$. Given increasing returns on skills and equal initial consumption in the resource-abundant country under trade and autarky, households in this country gain from trade in the long run.

Proposition 9 is illustrated by Fig. 4 where equilibrium dynamics are shown for country 2 under autarky and trade. The bold trajectory is an autarky equilibrium path, and the bold dashed trajectory is a trading equilibrium path. Both trajectories start from the same initial point (n_{02}, c_{02}) . The zero-investment loci are OA and OB for the autarky and the trading equilibrium, correspondingly. The long-term welfare gain results from two effects. On the one hand, curve OB locates above curve OA for all n_2 , because for any n_2 household income is higher in trade than in autarky: $q_2(n_2, X_2) > y(n_2, R_2)$, due to Proposition 6. On the other hand, the trading equilibrium path locates below the autarky equilibrium path because, given n_2 , consumption is lower in trading equilibrium. The

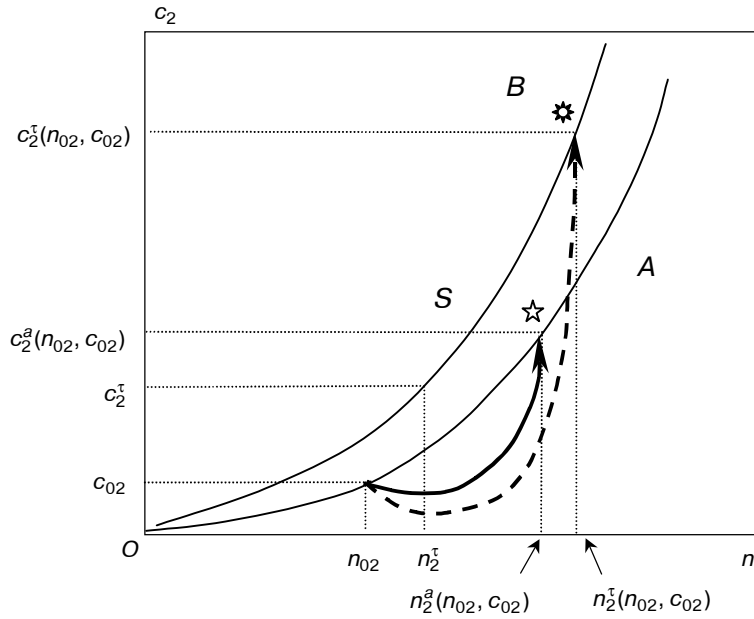


Figure 4. Equilibrium paths for country 2 under increasing returns.

return on skills in this country is lower under trading equilibrium: $\alpha y(n_2, X_2)/n_2 < \alpha y(n_2, R_2)/n_2$ because country 2 exports the raw material, and $X_2 < R_2$. Due to this, and because $h(c_2) = \text{const}$ we have that for any $n_2 > n_{02}$ the instantaneous increase of consumption dc_2/dt is lower in trading equilibrium. Correspondingly, for any $n_2 > n_{02}$ the increase of the number of skills dn_2/dt is higher when country 2 trades.

Fig. 4 suggests that economic growth continues in trading equilibrium, while it ceases in autarky when the steady state number of skills $n^a(n_{02}, c_{02})$ is reached. Consequently, the long-run number of skills $n^r(n_{02}, c_{02})$ and consumption $c^r(n_{02}, c_{02})$ are higher in trade than in autarky. The intuition of this result is that, under increasing returns, trade induces households in the resource-abundant country to reduce consumption at the early stage of development and to increase investment in skills. Country 2 accumulates more skills when it trades, and households in this country eventually gain from a higher income and consumption.

Note that in trading equilibrium country 2 may stop accumulating skills earlier than country 1. The constraint (12) in the household problem for country 2 becomes binding at some point in time, and consumption in this country becomes equal to income. At the same time equilibrium dynamics for country 1 is still governed by system (15), (16) with $j=1$ and $n_2 \equiv n_2(n_{02}, c_{02})$. Unlike the autarky case household income in country 2 continues to increase until the steady state is reached by country 1. This is so because $\partial q_2(n_2, X(n_1, n_2))/\partial n_1 > 0$, given that country 1 has comparative advantage in manufacturing.⁶ Therefore, accumulation of skills by the resource-scarce country favors the resource-abundant country. Households there continue to increase consumption without investment in skills and thus receive an additional long-term welfare gain from trade.

The welfare effect of trade on the resource-scarce country under increasing returns is ambiguous. A positive static effect on household income may be offset by a negative dynamic effect of raising the return on skills stimulating a transitory increase of consumption. As a result, the resource-scarce country may under-accumulate skills and reduce long-term welfare. A detailed analysis of this problem requires numerical simulation that is beyond the scope of this paper.

⁶ One can show that $\frac{\partial q_j}{\partial n_i} = (1 - \alpha)(\theta - 1)r_j \frac{X_i(R_j - X_j)}{(R_1 + R_2)X_j} > 0$, if $R_j > X_j$.

7. CONCLUDING REMARKS

The main result of the model is that the resource-abundant country can lose from international trade under decreasing returns on skills. The loss is positively linked to the absolute advantage of such a country in the traditional production. The resource-abundant country gains from trade under increasing returns on skills. Development paths of the countries converge in the case of decreasing returns but may diverge under increasing returns.

A policy implication of these results is that the resource-abundant country does not fall into an "underdevelopment trap" after opening up trade with advanced countries. The long-run losses from trade of this country under decreasing returns do not depend on the initial comparative advantages because initial conditions are irrelevant to long-term specialization in trade and trade patterns. From this view, policy measures aimed at "infant industry" protection are ineffective in the long-term perspective. The resource-abundant country does not need to create a sufficient industrial base before starting radical trade liberalization or to pursue a "preparatory" policy against unfavorable trade specialization through, *e.g.*, stimulating aggregate demand or raising real wages, as advocated for instance by Rodrik (1996). In our opinion, the state trying to diminish long-term welfare losses from trade should care about external effects of private investment. Households under decreasing returns on skills over-consume as compared to social optimum, because the social return on skills in this case is below the household return. By this reason an efficiency-improving policy should aim at creating a favorable investment regime and providing incentives for accumulation of skills.

APPENDIX

Proof of Proposition 1. The first-order condition for the household problem (1)–(3) is

$$u'(c) = v, \quad (\text{A1})$$

where v is the current value of the costate variable related to (2). The costate equation is

$$\frac{\dot{v}}{v} = \delta - \alpha(y(n, R)/n).$$

Suppose that the constraint (3) is non-binding. Then (A1) implies

$$\frac{\dot{v}}{v} = \dot{c}(u''(c)/u'(c))$$

and the co-state equation is written as (9).

The stationary point (n^a, c^a) is found directly from (8), (9). The characteristic equation for this point is

$$\begin{vmatrix} y'_n - \lambda & -1 \\ h(c^a)r'_n & -\lambda \end{vmatrix} = 0$$

where $y'_n = \partial y(n^a, R)/\partial n$, $r'_n = \alpha \partial(y(n^a, R)/n^a)/\partial n$. This is a second degree equation:

$$\lambda^2 - y'_n \lambda + h(c^a)r'_n = 0 \quad (\text{A2})$$

having two real-valued roots with opposite signs if $r'_n < 0$. Hence, the stationary point is a saddle if returns on skills are decreasing. The characteristic roots are complex with positive real part or positive real-valued if returns are increasing, that is $r'_n > 0$.

Proof of Proposition 2. The second equation in (14) implies that

$$\frac{R_2 + \Delta R}{R_1 - \Delta R} = \left(\frac{n_2}{n_1} \right)^{\theta-1}$$

which yields ΔR straightforwardly.

Proof of Corollary. The raw material input in country 1 is

$$X_1 = R_1 - \Delta R = R_1 \left(1 - \frac{(n_2 / n_1)^{\theta-1} - \rho}{1 + (n_2 / n_1)^{\theta-1}} \right) = \frac{R_1 + R_2}{1 + (n_2 / n_1)^{\theta-1}}.$$

Similarly,

$$\begin{aligned} X_2 &= R_2 + \Delta R = R_2 \left(1 + \frac{\rho^{-1}(n_2 / n_1)^{\theta-1} - 1}{1 + (n_2 / n_1)^{\theta-1}} \right) = \\ &= R_2 \left(1 + \frac{\rho^{-1} - (n_1 / n_2)^{\theta-1}}{1 + (n_1 / n_2)^{\theta-1}} \right) = \frac{R_1 + R_2}{1 + (n_1 / n_2)^{\theta-1}}. \end{aligned}$$

Consequently,

$$X_j = \frac{R_1 + R_2}{1 + (n_i / n_j)^{\theta-1}}$$

and $i \neq j$. The income in country 1 equals

$$\begin{aligned} q_1 &= y(n_1, X_1) + \rho \Delta R = y(n_1, X_1) + (1 - \alpha) \frac{y(n_1, X_1)}{X_1} \Delta R = \\ &= y(n_1, X_1) \left(1 + (1 - \alpha) \frac{\Delta R}{X_1} \right) = \\ &= y(n_1, X_1) \left(1 + (1 - \alpha) \frac{R_1 - X_1}{X_1} \right) = y(n_1, X_1) \left(\alpha + (1 - \alpha) \frac{R_1}{X_1} \right). \end{aligned}$$

Similarly,

$$\begin{aligned} q_2 &= y(n_2, X_2) - \rho \Delta R = y(n_2, X_2) \left(1 - (1 - \alpha) \frac{\Delta R}{X_2} \right) = \\ &= y(n_2, X_2) \left(1 - (1 - \alpha) \frac{X_2 - R_2}{X_2} \right) = y(n_2, X_2) \left(\alpha + (1 - \alpha) \frac{R_2}{X_2} \right). \end{aligned}$$

Proof of Proposition 3. Dynamic equilibrium equations (15), (16) are derived in the same way as equilibrium equations (8), (9) in the autarky case. We will demonstrate that the stationary point $S = (n_j^*, c_j^*)$ is unique in the proof of Proposition 4. Characteristic equation for system

(15), (16) in this point is

$$\begin{vmatrix} q_{11} - \lambda & -1 & q_{12} & 0 \\ h(c_1^\tau)r_{11} & -\lambda & h(c_1^\tau)r_{12} & 0 \\ q_{21} & 0 & q_{22} - \lambda & -1 \\ h(c_2^\tau)r_{21} & 0 & h(c_2^\tau)r_{22} & -\lambda \end{vmatrix} = 0.$$

Here

$$q_{ii} = \frac{\partial q_i(n_i^\tau, X_i(n_i^\tau, n_j^\tau))}{\partial n_i},$$

$$q_{ij} = \frac{\partial q_i(n_i^\tau, X_i(n_i^\tau, n_j^\tau))}{\partial n_j},$$

$$r_{ii} = \alpha \frac{\partial (y(n_i^\tau, X_i(n_i^\tau, n_j^\tau)) / n_i^\tau)}{\partial n_i},$$

$$r_{ij} = \alpha \frac{\partial (y(n_i^\tau, X_i(n_i^\tau, n_j^\tau)) / n_i^\tau)}{\partial n_j}.$$

Applying the Laplace theorem transforms this equation:

$$F_1(\lambda)F_2(\lambda) + F_{12}(\lambda) = 0, \quad (\text{A3})$$

where

$$F_1(\lambda) = \begin{vmatrix} q_{11} - \lambda & -1 \\ h(c_1^\tau)r_{11} & -\lambda \end{vmatrix} = \lambda^2 - q_{11}\lambda + h(c_1^\tau)r_{11},$$

$$F_2(\lambda) = \begin{vmatrix} q_{22} - \lambda & -1 \\ h(c_2^\tau)r_{22} & -\lambda \end{vmatrix} = \lambda^2 - q_{22}\lambda + h(c_2^\tau)r_{22},$$

$$\begin{aligned} F_{12}(\lambda) &= \begin{vmatrix} -1 & q_{12} \\ -\lambda & h(c_1^\tau)r_{12} \end{vmatrix} \times \begin{vmatrix} q_{21} & -1 \\ h(c_2^\tau)r_{21} & -\lambda \end{vmatrix} = \\ &= -q_{12}q_{21}\lambda^2 + (h(c_1^\tau)r_{12}q_{12} + h(c_2^\tau)r_{21}q_{21})\lambda - h(c_1^\tau)h(c_2^\tau)r_{12}r_{21}. \end{aligned}$$

As shown in Proposition 4, the long-run numbers of skills are identical across countries: $n_1^\tau = n_2^\tau$. Using this fact we can calculate q_{ii} , r_{ii} , q_{ij}

and $r_{ij} < 0$:

$$q_{ii} = (\theta - 1)r_i^\tau \left(1 + (1 - \alpha) \frac{(R_j - R_i)}{2(R_1 + R_2)} \right),$$

$$q_{ij} = (\theta - 1)r_i^\tau (1 - \alpha) \frac{(R_i - R_j)}{2(R_1 + R_2)},$$

$$r_{ii} = \frac{1}{2}(n_i^\tau)^{\alpha(\theta-1)-2} \left(\frac{R_1 + R_2}{2} \right)^{1-\alpha} \left(\frac{\alpha + \sigma}{\sigma - 1} - 3 \right),$$

$$r_{ij} = -\frac{1}{2}(1 - \alpha)(\theta - 1)(n_i^\tau)^{\alpha(\theta-1)-2} \left(\frac{R_1 + R_2}{2} \right)^{1-\alpha}.$$

Hence, $r_{ii} < 0$, if and only if $\sigma > (3 + \alpha)/2$.

Consider the term $F_1(\lambda)F_2(\lambda)$ on the right-hand side of (A3). It is a fourth-order polynomial having a couple of negative and a couple of positive real-valued roots. This is so because each equation $F_j(\lambda) = 0$, $j = 1, 2$, has two real-valued roots with opposite sign (each of these equations is similar to (A2)). The term $F_{12}(\lambda)$ is negative for all $\lambda < 0$, since $F'_{12}(0) > 0$. Adding this function to the term $F_1(\lambda)F_2(\lambda)$ maintains two negative characteristic roots if the intercept of equation (A3) is positive. This intercept equals to $h(c_1^\tau)h(c_1^\tau)(r_{11}r_{22} - r_{12}r_{21})$. It is positive since the returns on skills at the steady state satisfy: $r_{11} = r_{22}$, $r_{12} = r_{21}$, and $-r_{11} > -r_{12}$ for $\sigma > 1 + \alpha$. In fact, this is the case under decreasing returns, because $(3 + \alpha)/2 > 1 + \alpha$.

Proof of Proposition 4. Consider the steady state conditions

$$\dot{c}_j = 0, j = 1, 2.$$

These imply that

$$\frac{n_2}{n_1} = \frac{y(n_2, X_2)}{y(n_1, X_1)} = \left(\frac{n_2}{n_1} \right)^{\alpha(\theta-1)} \left(\frac{X_2}{X_1} \right)^{1-\alpha}.$$

From (14) the long-run numbers of skills satisfy

$$\frac{n_2}{n_1} = \left(\frac{n_2}{n_1} \right)^{\theta-1} \tag{A4}$$

This equation has a unique solution $n_2^\tau / n_1^\tau = 1$.

Consequently, $X_1^* = X_2^*$, $y(n_1^*, X_1^*) = y(n_2^*, X_2^*)$, and the steady state conditions $\dot{n}_j = 0$, $j = 1, 2$ imply that

$$\begin{aligned} \frac{c_2^*}{c_1^*} &= \frac{q(n_2^*, X_2^*)}{q(n_1^*, X_1^*)} = \frac{y(n_2^*, X_2^*)}{y(n_1^*, X_1^*)} \frac{(\alpha + (1-\alpha)R_2 / X_2^*)}{(\alpha + (1-\alpha)R_1 / X_1^*)} = \\ &= \frac{\alpha(R_1 + R_2) + (1-\alpha)2R_2}{\alpha(R_1 + R_2) + (1-\alpha)2R_1} = \frac{\alpha(1+\rho) + (1-\alpha)2\rho}{\alpha(1+\rho) + (1-\alpha)2}. \end{aligned}$$

Proof of Proposition 5. Taking derivatives yields

$$\frac{\partial \dot{n}_j}{\partial n_j} = \alpha(\theta - 1)y(n_j, R_j)n_j^{-1}$$

under autarky, and

$$\begin{aligned} \frac{\partial \dot{n}_j}{\partial n_j} &= y(n_j, X_j) \times \left[\left(\alpha(\theta - 1)n_j^{-1} + (1-\alpha)(\theta - 1) \frac{X_i}{R_1 + R_2} n_j^{-1} \right) \left(\alpha + (1-\alpha) \frac{R_j}{X_j} \right) - \right. \\ &\quad \left. - (1-\alpha)(\theta - 1) \frac{R_j}{R_1 + R_2} \frac{X_i}{X_j} n_j^{-1} \right] = \\ &= y(n_j, X_j) \frac{\alpha(\theta - 1)}{n_j} \left[1 + (1-\alpha) \frac{X_i}{R_1 + R_2} \left(1 - \frac{R_j}{X_j} \right) \right] \end{aligned}$$

under trade. Hence, $\Delta^* > \Delta^a$ if

$$\Delta_{j=1,2} \frac{y(n_j, X_j) - y(n_j, R_j)}{n_j} + (1-\alpha) \Delta_{j=1,2} \frac{y(n_j, X_j)}{n_j} \frac{X_i}{R_1 + R_2} \left(1 - \frac{R_j}{X_j} \right) > 0, \quad (\text{A5})$$

where symbol Δ refers to difference between variables related to country 1 and country 2, correspondingly. The first term on the left-hand side of this inequality transforms to:

$$\begin{aligned} \Delta_{j=1,2} \frac{y(n_j, X_j) - y(n_j, R_j)}{n_j} &= \Delta_{j=1,2} \frac{y(n_j, X_j)}{n_j} \left[1 - (R_j / X_j)^{1-\alpha} \right] > \\ &> \frac{y(n_j, X_j)}{n_j} \left[1 - \alpha - (1-\alpha) (R_j / X_j) \right] = (1-\alpha) \Delta_{j=1,2} \frac{y(n_j, X_j)}{n_j X_j} (X_j - R_j). \end{aligned} \quad (\text{A6})$$

One can show that

$$\frac{y(n_1, X_1)}{y(n_2, X_2)} = \frac{X_1}{X_2} = \left(\frac{n_1}{n_2} \right)^{\theta-1}. \quad (\text{A7})$$

Therefore (A6) is positive if

$$(X_1 - R_1) - \frac{n_1}{n_2} (X_2 - R_2) = (X_1 - R_1) \left(1 + \frac{n_1}{n_2} \right) > 0.$$

This is the case if $X_1 > R_1$, or equivalently, country 1 has comparative advantage in manufacturing.

Consider the second term on the left-hand side of (A5). It is positive if

$$\Delta_{j=1,2} \frac{y(n_j, X_j) X_j}{n_j X_j} (X_j - R_j) > 0$$

Due to (A7) the left-hand side of this inequality is equivalent to:

$$\begin{aligned} (X_1 - R_1) \frac{y(n_1, X_1) X_2}{n_1 X_1} \left(1 + \frac{y(n_2, X_2) X_1^2 n_1}{y(n_1, X_1) X_2^2 n_2} \right) &= \\ &= (X_1 - R_1) \frac{y(n_1, X_1) X_2}{n_1 X_1} \left(1 + \frac{X_1 n_1}{X_2 n_2} \right) = (X_1 - R_1) \frac{y(n_1, X_1) X_2}{n_1 X_1} \left(1 + \left(\frac{n_1}{n_2} \right)^{\theta} \right). \end{aligned}$$

It is positive if $X_1 > R_1$.

Proof of Proposition 6. We want to demonstrate that

$$n_j^{\alpha(\theta-1)} X_j^{1-\alpha} (\alpha + (1-\alpha) R_j / X_j) > n_j^{\alpha(\theta-1)} R_j^{1-\alpha}. \quad (\text{A8})$$

This is equivalent to

$$(\alpha + (1-\alpha) R_j / X_j) > (R_j / X_j)^{1-\alpha},$$

which holds for all n_j such that $R_j \neq X_j$.

Proof of Proposition 7. The steady state conditions $\dot{c}_j = 0$

imply that

$$n_j^a = \left(\frac{\alpha}{\delta} \right)^{\frac{1}{1-\alpha(\theta-1)}} R_j^\beta, \quad n_j^\tau = \left(\frac{\alpha}{\delta} \right)^{\frac{1}{1-\alpha(\theta-1)}} (X_j^\tau)^\beta.$$

Hence,

$$\frac{n_j^\tau}{n_j^a} = \left(\frac{X_j^\tau}{R_j} \right)^\beta$$

and, from Corollary to Proposition 2 and Proposition 4,

$$\frac{n_1^\tau}{n_1^a} = \left(\frac{1+\rho}{2} \right)^\beta, \quad \frac{n_2^\tau}{n_2^a} = \left(\frac{1+\rho}{2\rho} \right)^\beta.$$

Proof of Proposition 8. Country 1 gains from trade due to Propositions 6 and 7:

$$q(n_1^\tau, X_1^\tau) > q(n_1^a, X_1^\tau) > y(n_1^a, R_1^\tau).$$

Hence,

$$c_1^\tau > c_1^a.$$

Consider the same condition for country 2:

$$c_2^\tau > c_2^a.$$

It holds if

$$\begin{aligned} q(n_2^\tau, X_2^\tau) &= \left(\frac{1+\rho}{2\rho} \right)^{\alpha(\theta-1)\beta} (n_2^a)^{\alpha(\theta-1)} (X_2^\tau)^{1-\alpha} (\alpha + (1-\alpha)R_2 / X_2^\tau) > \\ &> (n_2^a)^{\alpha(\theta-1)} (R_2)^{1-\alpha} = y(n_1^a, R_2) \end{aligned}$$

due to Proposition 7. Hence, country 2 benefits from trade if

$$\left(\frac{1+\rho}{2\rho} \right)^{\alpha(\theta-1)\beta} \left(\frac{X_2^\tau}{R_2} \right)^{1-\alpha} \left(\alpha + (1-\alpha) \frac{R_2}{X_2^\tau} \right) > 1$$

or, equivalently,

$$\left(\frac{1+\rho}{2\rho} \right)^{\alpha(\theta-1)\beta} \left(\frac{1+\rho}{2\rho} \right)^{1-\alpha} \left(\alpha + (1-\alpha) \frac{2\rho}{1+\rho} \right) > 1.$$

This can be written as

$$\left(\frac{1+\rho}{2\rho}\right)^\beta \left(\alpha + (1-\alpha)\frac{2\rho}{1+\rho}\right) > 1, \quad (\text{A9})$$

which is equivalent to (18). This inequality does not hold for all $\rho > 1$ if $\beta \geq 1$, or $\sigma \leq 2$. Hence, country 2 loses from trade if $\sigma < 2$.

Proof of Proposition 9 is contained in the elucidation to Fig. 4.

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